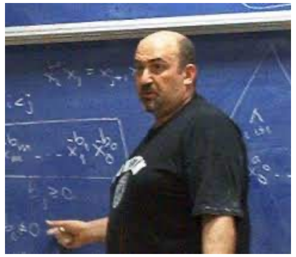
Geometric, Algebraic and Probabilistic Combinatorics (GAPCOMB)

Universidad Politécnica de Cataluña

People

FACULTY

Simeon Ball



Josep Burillo

Anna de Mier

Marc Noy

Guillem Perarnau

Julian Pfeifle

Juanjo Rué

Oriol Serra

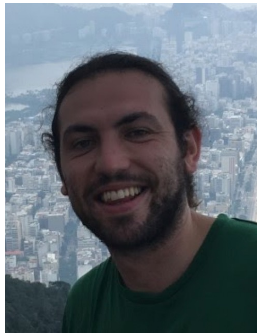
Lluís Vena

Enric Ventura

POSTDOCS

Jordi Delgado

María Zambrano (E. Ventura)



Patrick Morris

Marie Curie (G. Perarnau)

Richard Lang

Ramón y Cajal (G. Perarnau)

Tássio Naia

María de Maeztu (G. Perarnau)

Clément Requilé

Beatriu de Pinós (M. Noy)

Malika Roy

Margarita Salas (E. Ventura, UPV-UPC)

PHD STUDENTS

Sofiya Burova (with UPF)

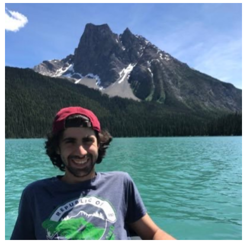
UPF (G. Lugosi-G. Perarnau)

Jordi Castellví

FPI (M. Noy-C. Requilé)

Miquel Ortega

FPI (O. Serra)

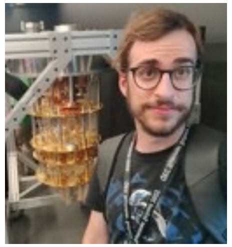


Tabriz Popatia

UPC (S. Ball)

Robin Simoens (with Ghent U)

Belgium Agency (A. Abiad-S. Ball-L. Storme)



Ricard Vilar

UPC (S. Ball)

Collaborations

Red DAM

U Autónoma Madrid
U Cantabria
U Pompeu Fabra
U Sevilla

OTHERS

Charles U Prague
Rényi Institute Budapest
TU Berlin
TU Vienna
TU Warsaw
U Andrés Bello, Chile
U Bordeaux
U Paris Cité

Related groups at UPC

- Algorithmics, Bioinformatics, Complexity and Formal Methods

  A. Atserias, I. Bonacina, J. Díaz, M.J. Serna

- Combinatorics, Graph Theory, and Applications

  F. Comellas, J. Fàbrega, M.A. Fiol, J. Martí, J. Muñoz, S. Pérez

- Discrete, Combinatorial and Computational Geometry

  C. Hernando, C. Huemer, F. Klute, M. Mora, I. Parada, C. Seara, R. Silveira

- Mathematics Applied to Crytpography

  J. Herranz, P. Morillo, C. Padró, G. Sáez, J. Villar

- Matrix Analysis and Discrete Potential Theory

  A. Carmona, A. Encinas, M.J. Jiménez, M. Mitjana, E. Monso

Activities

Weekly seminar on Combinatorics, Graph Theory, Algorithmics and Theory of Computation (Thursdays 16:15, Broadcasted online)

2023

January	Robin Simoens (Ghent U), Alexandra Wesolek (Simon Fraser U)
February	Bas Lodewijks (U Lyon), Christoph Spiegel (Zuse I. Berlin), Oriol Serra (UPC), Patrick Morris (UPC)
March	Alberto Larrauri (TU Graz), Arnau Padrol (UB), Alberto Espuny (TU Ilmenau), Guillem Perarnau (UPC), Miquel Ortega (UPC)
April	Dimitrios Thilikos (CNRS Montpellier), Kilian Rothmund (U Ulm), Amarja Kathapurkar (U Birmingham), Simeon Ball and Ricard Vilar (UPC)
May	Giovanne Santos (U Chile), Clément Requilé (UPC)
June	Sam Mattheus (UC San Diego), Xavier Povill (UPC)
July	Alp Müyesser (U College London), Vasiliki Velona (Hebrew U Jerusalem)
September	Benedikt Stufler (TU Vienna), Patrick Morris (UPC)
October	Rui Zhang (UPF), Suchismita Mishra (U. A. Bello, Chile), Marcos Kiwi (U. Chile)
November	Amanda Montejano (UNAM México), Mehmet Akif Yildiz (U Amsterdam), Xavier Pérez (U Nebraska), Tássio Naia (CRM) Fionn Mc Inerney (TU Vienna)
December	Oleg Pikhurko (U Warwick)

GAPCOMB Workshop

Annual meeting of the group, held in July since 2019 (in Montserrat since 2021, no edition in 2020)

Devoted to problem solving since (2022) with members of the group, guests, and master students

2022 (20 participants)

2023 (26 participants)

Reading Seminar

Devoted to reviewing recent results in combinatorics

October 2022 – January 2023
Proof of the Kahn-Kalai conjecture on thresholds of monotone properties on random graphs
(by Jinyoung Park and Huy Tuan Pham)

Expositions (7) by members of the group and discussions (2) on related results

April-July 2023
An exponential improvement for diagonal Ramsey numbers
(by Marcelo Campos, Simon Griffiths, Robert Morris, and Julian Sahasrabudhe)

Following online lectures by Rob Morris (available on YouTube)

Grants

- PID2020 Combinatorics: new trends and real-world applications

  PI: Simeon Ball, Guillem Perarnau

- PID2021 Geometric methods in group theory

  PI: Enric Ventura

- UPC Support to research groups 2022, 2023

  PI: Simeon Ball

Marie Curie Research and Innovation Staff Exchange Programme (RISE)

RandNET Grant on Randomness and Learning in Networks (2021-2026). PI: M. Noy

Europe

UPC (coordinator)
UPF
Charles U Prague
École Polytechnique
TU Eindhoven
TU Vienna
U Oxford
U Paris-Cité
Nokia Bell Labs France

Overseas

Georgia Tech
IMPA Rio de Janeiro
McGill U
U Chile
UC San Diego

Funding for research stays (of at least one month) from EU partners to overseas

22-30 August 2022 Eindhoven: RandNET Summer School & Workshop on Random Graphs

13-16 September 2023 Prague: Workshop on Graph Limits and Networks

August 2024 Rio de Janeiro: Summer School & Workshop on Learning and Combinatorial Statistics

2025 Vienna: Workshop on Combinatorial Parameters of Random Graphs and Algorithms

Research topics

- Combinatorial and geometric group theory (Cayley graphs, free groups)
- Combinatorial number theory (sum-free sets, Sidon sets)
- Enumerative combinatorics (enumeration of planar maps and graphs, graphs with given tree-width)
- Extremal combinatorics (graph orientations, Ramsey theory)
- Finite geometries and Coding theory (quantum error-correcting codes, MDS codes)
- Matroids, Polytopes and Graph polynomials (Graph polynomials, polytope realizability)
- Random graphs and random discrete structures (random graphs and digraphs, percolation, graph coloring)

Voting systems and analytic combinatorics

Emma Caizergues, François Durand, Élie de Panafieu
Nokia Bell Labs France

Vlady Ravelomanana Université Paris Cité

Marc Noy UPC Barcelona

Voting settings

- m = number of candidates, n = number of voters
- Each voter has strict preferences over the candidates

Elections in Australia: Instant-runoff voting

A candidate is a Condorcet winner if she/he is preferred to every other candidate by the majority rule

A first example

Thanks to Emma Caizergues for the pictures

1	5	0	6	1	3	= 16
a	a	b	b	c	c	
b	c	a	c	a	b	
c	b	c	a	b	a	

$$c \succ a: \quad 10 > 16/2$$

Figure: 3 candidates, 16 voters

A first example

Thanks to Emma Caizergues for the pictures

1	5	0	6	1	3	$= 16$
a	a	b	b	c	c	
b	c	a	c	a	b	
c	b	c	a	b	a	

$c \succ a$: $10 > 16/2$

Figure: 3 candidates, 16 voters

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a	a	b	b	c	c	
b	c	a	c	a	b	
c	b	c	a	b	a	

$c \succ a$: $10 > 16/2$

Figure: 3 candidates, 16 voters

The Condorcet paradox

- A Condorcet winner is a candidate who wins all pairwise comparisons
- The Condorcet paradox occurs when there is no Condorcet winner

1	1	1
a	b	c
b	c	a
c	a	b

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1	1	1
a	b	c
b	c	a
c	a	b

$$a \succ b$$
$$b \succ c$$
$$c \succ a$$

Framework : General Independent Culture

A culture is a probability distribution on possible orderings

p_1	p_2	p_3	p_4	p_5	p_6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

$$p_1 + \cdots + p_6 = 1$$

Particular case : Impartial Culture

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

Some results

Let $\mathbb{P}_{n,m}(CW)$ be the probability that here is a Condorcet winner (under impartial culture) with n voters and m candidates

$\lim_{n\to\infty} \mathbb{P}_{n,2}(CW) = 1$

Some results

Let $\mathbb{P}_{n,m}(CW)$ be the probability that here is a Condorcet winner (under impartial culture) with n voters and m candidates

$\lim_{n\to\infty} \mathbb{P}_{n,2}(CW) = 1$

$\lim_{n\to\infty} \mathbb{P}_{n,3}(CW) = \frac{3}{4} + \frac{3}{2\pi} \arcsin(1/3) \approx 0.91$

Enter generating functions

Computing the probablity that **c** is a Condorcet winner

p_1	p_2	p_3	p_4	p_5	p_6
a	a	b	b	$\boxed{c}$	$\boxed{c}$
b	$\boxed{c}$	a	$\boxed{c}$	a	b
$\boxed{c}$	b	$\boxed{c}$	a	b	a
$x_a x_b$	x_a	$x_a x_b$	x_b	1	1

$x_a \leftrightarrow a > c$

$x_b \leftrightarrow b > c$

Enter generating functions

Computing the probablity that **c** is a Condorcet winner

p_1	p_2	p_3	p_4	p_5	p_6
a	a	b	b	$\boxed{c}$	$\boxed{c}$
b	$\boxed{c}$	a	$\boxed{c}$	a	b
$\boxed{c}$	b	$\boxed{c}$	a	b	a
$x_a x_b$	x_a	$x_a x_b$	x_b	1	1

$x_a \leftrightarrow a > c$

$x_b \leftrightarrow b > c$

$$P(x_a, x_b) = p_5 + p_6 + p_2 x_a + p_4 x_b + (p_1 + p_3) x_a x_b$$

Coefficient extraction

Given $A(x) = \sum_{k \geq 0} a_k x^k$ set

$$[x^n]A(x) = a_n \qquad [x^{\leq n}]A(x) = a_0 + a_2 + \cdots + a_n = [x^n]\frac{A(x)}{1-x}$$

The probability that a is preferred to c exactly k times and b is preferred to c exactly ℓ times is

$$[{x_a}^k {x_b}^\ell](P(x_a, x_b))^n$$

Coefficient extraction

Given $A(x) = \sum_{k \geq 0} a_k x^k$ set

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The probability that a is preferred to c exactly k times and b is preferred to c exactly ℓ times is

$$[{x_a}^k {x_b}^\ell](P(x_a, x_b))^n$$

The probability that the last candidate c is a Condorcet Winner is

$$\left[{x_a}^{\leq n/2} {x_b}^{\leq n/2}\right](P(x_a, x_b))^n = \left[{x_a}^{n/2} {x_b}^{n/2}\right] \frac{(P(x_a, x_b))^n}{(1-x_a)(1-x_b)}$$

Case m arbitrary

n voters and m candidates

$\boldsymbol{x} = (x_1, \ldots, x_{m-1})$

The probability that candidate m is a Condorcet winner is

$$\mathbb{P}_{n,m}(CW) = \left[\boldsymbol{x}^{n/2}\right] \frac{(P(\boldsymbol{x}))^n}{\prod_{k=1}^{m-1}(1 - x_k)}$$

Case m arbitrary

n voters and m candidates
$\boldsymbol{x} = (x_1, \ldots, x_{m-1})$
The probability that candidate m is a Condorcet winner is

$$\mathbb{P}_{n,m}(CW) = \left[\boldsymbol{x}^{n/2}\right] \frac{(P(\boldsymbol{x}))^n}{\prod_{k=1}^{m-1}(1-x_k)}$$

$$\mathbb{P}_{n,m}(CW) = \frac{1}{(2i\pi)^{m-1}} \oint \frac{(P(\boldsymbol{x}))^n}{\prod_{k=1}^{m-1}(1-x_k)} \frac{d\boldsymbol{x}}{\prod_{k=1}^{m-1} x_k^{n/2+1}}.$$

Case m arbitrary

n voters and m candidates
$\boldsymbol{x} = (x_1, \ldots, x_{m-1})$
The probability that candidate m is a Condorcet winner is

$$\mathbb{P}_{n,m}(CW) = \left[\boldsymbol{x}^{n/2}\right] \frac{(P(\boldsymbol{x}))^n}{\prod_{k=1}^{m-1}(1 - x_k)}$$

$$\mathbb{P}_{n,m}(CW) = \frac{1}{(2i\pi)^{m-1}} \oint \frac{(P(\boldsymbol{x}))^n}{\prod_{k=1}^{m-1}(1 - x_k)} \frac{d\boldsymbol{x}}{\prod_{k=1}^{m-1} x_k^{n/2+1}}.$$

$$= \frac{1}{(2i\pi)^{m-1}} \oint e^{n\left(log(P(\boldsymbol{x}) - \frac{1}{2}\sum_{k=1}^{m-1} x_k\right) + \sum_{k=1}^{m-1} \frac{1}{1-x_k}} \frac{d\boldsymbol{x}}{\boldsymbol{x}}$$

Saddle point method

$I_n = \int_{M \subseteq \mathbb{C}^d} A(\boldsymbol{x}) e^{n\phi(\boldsymbol{x})} d\boldsymbol{x}$, A and ϕ analytic

If ϕ has a unique critical point $\mathbf{0}$ on $\mathcal{M}$ then

$$I_n \underset{n\to\infty}{\sim} (2\pi n)^{-d/2} A(\mathbf{0}) e^{n\phi(\mathbf{0})} \det(\mathcal{H}(\mathbf{0}))^{-1/2}$$

$$\mathbb{P}_{n,m}(CW) = \frac{1}{(2i\pi)^{m-1}} \oint e^{n\left(log(P(\boldsymbol{x}) - \frac{1}{2}\sum_{k=1}^{m-1} \log(x_k)\right) + \sum_{k=1}^{m-1} \log(\frac{1}{1-x_k})} \frac{d\boldsymbol{x}}{\boldsymbol{x}}$$

One of our contributions: subcritical case

- $\mathbb{P}_{n,m}(CW)$ = probability last candidate is a Condorcet winner
- $\phi(\boldsymbol{x}) = \log(P(\boldsymbol{x})) - \frac{1}{2}\sum_{k=1}^{m-1}\log(x_k)$
- $\boldsymbol{\zeta}$ = solution to $(\partial_k \phi(\boldsymbol{x}) = 0)_k$

If $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_{m-1})$ with $\zeta_j < 1$ for all j then

$$\mathbb{P}_{n,m}(CW) \underset{n\to\infty}{\sim} \frac{1}{\sqrt{(2\pi n)^{m-1}}} \prod_{k=1}^{m-1} \frac{\zeta_k^{-n/2}}{1-\zeta_k} \frac{e^{n\phi(\mathbf{1})}}{\sqrt{\det(\mathcal{H}_\phi(\mathbf{1}))}}$$

The arc-sinus formula revisited

3 candidates and $2n+1$ voters, impartial culture

$$\mathbb{P}(3,\infty) = 3[x^n y^n] \frac{\frac{1}{6}(2+x+y+2xy)^{2n+1}}{(1-x)(1-y)}$$

$$M = \begin{pmatrix} 1/4 & 1/12 \\ 1/12 & 1/4 \end{pmatrix}$$

$$\mathbb{P}(3,\infty) = 3\left(\frac{i}{2\pi}\right)^2 \int_{(i-\infty, i+\infty)^2} e^{-\frac{1}{2}(x\ y)M\begin{pmatrix} x \\ y \end{pmatrix}} \frac{dxdy}{xy}$$

$$\mathbb{P}(3,\infty) = 3\left(\frac{i}{2\pi}\right)^2 \int_{(i-\infty, i+\infty)^2} e^{-\frac{1}{2}(x \;\; y) M \begin{pmatrix} x \\ y \end{pmatrix}} \frac{dxdy}{xy}$$

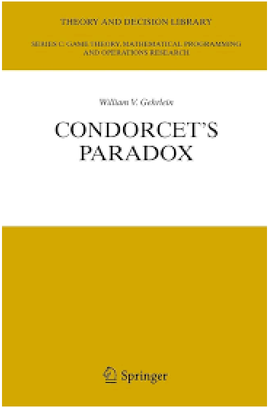
$$= -\frac{3}{4\pi^2} \int_{(i-\infty, i+\infty)^2} \sum_{k\geq 0} \frac{(-1/12)^k}{k!} (xy)^{k-1} e^{-x^2/8 - y^2/8} dxdy$$

$$= -\frac{3}{4\pi^2} \sum_{k\geq 0} \frac{(-1/12)^k}{k!} \left(\int_{i-\infty}^{i+\infty} x^{k-1} e^{-x^2/8} dx\right)^2$$

$$= \frac{3}{4} + \frac{3}{2\pi} \sum_{j\geq 0} \frac{(1/3)^{2j+1}}{(2j+1)!} \left(\frac{(2j)!}{2^j j!}\right)^2$$

$$\mathbb{P}(3,\infty) = \frac{3}{4} + \frac{3}{2\pi} \arcsin\left(\frac{1}{3}\right)$$

Musée Carnavalet Paris



W. Gehrlein (U. Delaware) 2006

2023/24-01:FME-200003-CU | Discrete Mathematics Days 2

https://dmd2024.web.uah.es

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Discrete Mathematics Days 2024

Alcalá de Henares, July 3 - 5, 2024

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Committees
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Previous editions
Ramon Llull prize

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Introduction

The Discrete Mathematics Days (DMD 2024) will be held on July 3-5, 2024, at the Universidad de Alcalá, in Alcalá de He
The main focus of this international conference is on current topics in Discrete Mathematics, including (but not limited t

- Coding Theory and Cryptography.
- Combinatorial Number Theory.
- Combinatorics.
- Discrete and Computational Geometry.
- Discrete Optimization

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